Hype for Types
HW 07
Due 15 October 2019

Question 1

Recall from lecture that we defined a *model* of DPDL to be a set X equipped with partial functions $\|\pi\| : X \rightharpoonup X$ for each $\pi \in \Pi$ (where Π was our set of programs), and for each propositional letter p, a set $[\![p]\!] \subseteq X$, the *extension* of p, or the set of "all those states $x \in X$ where p is true". We then recursively defined the extensions for various logical connectives:

$$\begin{aligned}
x \in [\![\neg\varphi]\!] &\iff x \notin [\![\varphi]\!] \\
x \in [\![\varphi \wedge \psi]\!] &\iff x \in [\![\varphi]\!] \cap [\![\psi]\!] \\
x \in [\![\varphi \to \psi]\!] &\iff x \notin [\![\varphi]\!] \text{ or } x \in [\![\psi]\!] \\
x \in [\![\, [\pi]\varphi \,]\!] &\iff \|\pi\|(x) \in [\![\varphi]\!] \text{ or } \|\pi\|(x) \text{ is undefined} \\
x \in [\![\, \langle\pi\rangle\varphi \,]\!] &\iff x \in [\![\neg[\pi]\neg\varphi]\!]
\end{aligned}$$

So: what has to be true about the partial function $\|\pi\|$ at a state x in order for it to be the case that $x \in [\![\, [\pi]\varphi \,]\!]$ but $x \notin [\![\, \langle\pi\rangle\varphi \,]\!]$? Why?

Question 2

Annotate the following piece of C-like code with the appropriate Hoare Logic annotations to prove that it correctly computes $n!$. Assume `n` is given some integer value (but put in the correct precondition!). Like the example done in lecture, the final { } doesn't need to literally say that `res` = `n!`, but it should clearly imply it.

```
{            }
i:=n;
res:=1;
 {                  }
 while (i>0) do (
    {                        }
    res := res * i;
    i := i-1
    {                        }
)
{                                    }
```
