98-317 Hype for Types

Lecture 2

Algebraic Data Types

Introduction

Today in class we looked at a small language called E♭ which introduced us to sums and products. These notes contain some example proof trees of typechecking E♭ as well as the full grammar and statics of E♭.

Typechecking Examples

$$\dfrac{\dfrac{}{\cdot \vdash 1 : \texttt{int}}\;\textsf{int} \qquad \dfrac{}{\cdot \vdash \texttt{"hype"} : \texttt{str}}\;\textsf{str}}{\cdot \vdash (1, \texttt{"hype"}) : \texttt{int} \times \texttt{str}}\;\textsf{pair}$$

$$\dfrac{\dfrac{\dfrac{}{x : \texttt{int} \vdash x : \texttt{int}}\;\textsf{var} \qquad \dfrac{}{x : \texttt{int} \vdash \texttt{"types"} : \texttt{str}}\;\textsf{str}}{x : \texttt{int} \vdash (x, \texttt{"types"}) : \texttt{int} \times \texttt{str}}\;\textsf{pair}}{x : \texttt{int} \vdash \pi_2(x, \texttt{"types"}) : \texttt{str}}\;\textsf{rproj}$$

$$\dfrac{\dfrac{\dfrac{}{\cdot \vdash 1 : \texttt{int}}\;\textsf{int} \qquad \dfrac{}{\cdot \vdash \texttt{"hype"} : \texttt{str}}\;\textsf{str}}{\cdot \vdash (1, \texttt{"hype"}) : \texttt{int} \times \texttt{str}}\;\textsf{pair}}{\cdot \vdash \pi_1(1, \texttt{"hype"}) : \texttt{int}}\;\textsf{lproj}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot \vdash 1 : \texttt{int}}\;\textsf{int} \qquad \dfrac{}{\cdot \vdash \texttt{"5"} : \texttt{str}}\;\textsf{str}}{\cdot \vdash (1, \texttt{"5}) : \texttt{int} \times \texttt{str}}\;\textsf{pair} \qquad \dfrac{}{\cdot \vdash 0 : \texttt{int}}\;\textsf{int}}{\cdot \vdash ((1, \texttt{"5"}), 0) : (\texttt{int} \times \texttt{str}) \times \texttt{int}}}{\cdot \vdash \pi_1((1, \texttt{"5"}), 0) : \texttt{int} \times \texttt{str}}\;\textsf{lproj}}{\cdot \vdash \pi_2\ \pi_1((1, \texttt{"5"}), 0) : \texttt{int}}\;\textsf{rproj}$$

$$\dfrac{\cdot \vdash \texttt{"hello"} : \texttt{str}}{\cdot \vdash \texttt{INL "hello" as str} + \texttt{int} : \texttt{str} + \texttt{int}}\;\textsf{inl}$$

Grammar

Old/uninteresting cases have been grayed out.

Type	τ	::=	`int`	
		|	`str`	
		|	$\tau + \tau$	sum type
		|	$\tau \times \tau$	product type

Expression	e	::=	`x`	variable
		|	$\overline{n}$	integer literal
		|	$\overline{s}$	string literal
		|	`let` $x = e_1$ `in` e_2	
		|	(e_1, e_2)	pair
		|	$\pi_1\ e$	left projection
		|	$\pi_2\ e$	right projection
		|	`INL` e `as` $\tau_1 + \tau_2$	left injection
		|	`INR` e `as` $\tau_1 + \tau_2$	right injection
		|	`case` e `of` `INL` $x_1 \Rightarrow e_1 \mid$ `INR` $x_2 \Rightarrow e_2$	case analysis

Statics

Old/uninteresting rules have been grayed out.

$$\frac{}{\Gamma, x : \tau \vdash x : \tau}\ \textsf{var} \qquad \frac{}{\Gamma \vdash \overline{n} : \texttt{int}}\ \textsf{int} \qquad \frac{}{\Gamma \vdash \overline{s} : \texttt{str}}\ \textsf{str}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma, \texttt{let}\ x = e_1\ \texttt{in}\ e_2 : \tau_2}\ \textsf{let} \qquad \frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash (e_1, e_2) : \tau_1 \times \tau_2}\ \textsf{pair}$$

$$\frac{\Gamma \vdash e : \tau_1 \times \tau_2}{\Gamma \vdash \pi_1 e : \tau_1}\ \textsf{lproj} \qquad \frac{\Gamma \vdash e : \tau_1 \times \tau_2}{\Gamma \vdash \pi_2 e : \tau_2}\ \textsf{rproj}$$

$$\frac{\Gamma \vdash e : \tau_1}{\Gamma \vdash \texttt{INL}\ e\ \texttt{as}\ \tau_1 + \tau_2 : \tau_1 + \tau_2}\ \textsf{inl} \qquad \frac{\Gamma \vdash e : \tau_2}{\Gamma \vdash \texttt{INR}\ e\ \texttt{as}\ \tau_1 + \tau_2 : \tau_1 + \tau_2}\ \textsf{inr}$$

$$\frac{\Gamma \vdash e : \tau_1 + \tau_2 \quad \Gamma, x_1 : \tau_1 \vdash e_1 : \tau \quad \Gamma, x_2 : \tau_2 \vdash e_2 : \tau}{\Gamma \vdash \texttt{case}\ e\ \texttt{of}\ \texttt{INL}\ x_1 \Rightarrow e_1 \mid \texttt{INR}\ x_2 \Rightarrow e_2 : \tau}\ \textsf{case}$$